EELE 477
Digital Signal Processing

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Spectrum Analysis

Spectral Analysis

- Here is the problem: *we are given a discrete-time signal (or a portion of the signal) and we would like to know its frequency content. In other words, we want to represent it using a sum of complex exponentials.*
- Question: *can we compute the magnitude and phase of the complex exponentials given only the discrete-time sequence?*

Frequency Spectrum

- For this chapter, we define:

$$x[n] = X_0 + \sum_{k=1}^{N} \left(X_k e^{j\hat{\omega}_k n} + X_k^* e^{-j\hat{\omega}_k n} \right)$$

where $X_k = A_k e^{j\phi_k}$

- We want to find the X_k phasors that comprise the spectrum
- Note again that the spectrum is periodic and has both positive and negative frequency components

Spectral Viewpoint

- The spectrum is periodic in 2π, so any span of 2π is enough to know the whole spectrum
- We have often used the span $-\pi$ to π as the 2π range for convenient sketches
- Now, for the discrete Fourier transform (DFT), it is helpful to use span 0 to 2π

Discrete Fourier Transform

- Fourier analysis expression:

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j(2\pi nk/N)}, \quad \mathrm{k} = 0, 1, 2, \ldots, \mathrm{N}-1$$

- Characteristics:
  - Discrete and finite length (*N*) input x[n]
  - Discrete and finite length (*N*) output X[k]
  - X[k] are generally complex even if x[n] real

Compare to DTFT

- Recall the discrete-time Fourier transform of a finite-length sequence:

$$Y(\hat{\omega}) = \sum_{k=0}^{N-1} y[k] e^{-j\hat{\omega}k}$$

- If we *sample* the output X(), i.e., let

$$\hat{\omega} = \frac{2\pi k}{N},\ k = 0, 1, ..., N - 1$$

DFT Interpretation

- Therefore, the DFT X[k] corresponds to *N* equally spaced samples of $X(\omega)$ from 0 to 2π
- Another viewpoint:
  - Start with x[n]
  - Create N complex exponential sequences of length N, frequency $2\pi k/N$
  - Multiply x[n] by each exponential sequence, then sum each over the N samples

DFT Interpretation (cont.)

- It is possible to view DFT as a modulation+filtering system:
  - Each output X[k] is obtained by modulating the input sequence by $\exp(-j2\pi nk/N)$
  - The resulting modulated N-point sequence is then filtered with a N-point running sum (summed over 0 to N-1)