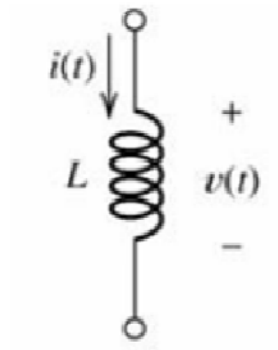
REVIEW OF INDUCTORS AND CAPACITORS AND RL CIRCUITS

Circuits with DC sources, Resistances and a Single Inductance

Review of Inductors



- Select all answers that apply to inductors:

  i) I = L dV/dt

  ii) V = L dI/dt

  iii) Voltage cannot change instantaneously in an inductor

  iv) Current cannot change instantaneously in an inductor

  v) Store energy in an electric field

  a) ii, iv

  b) i, iii

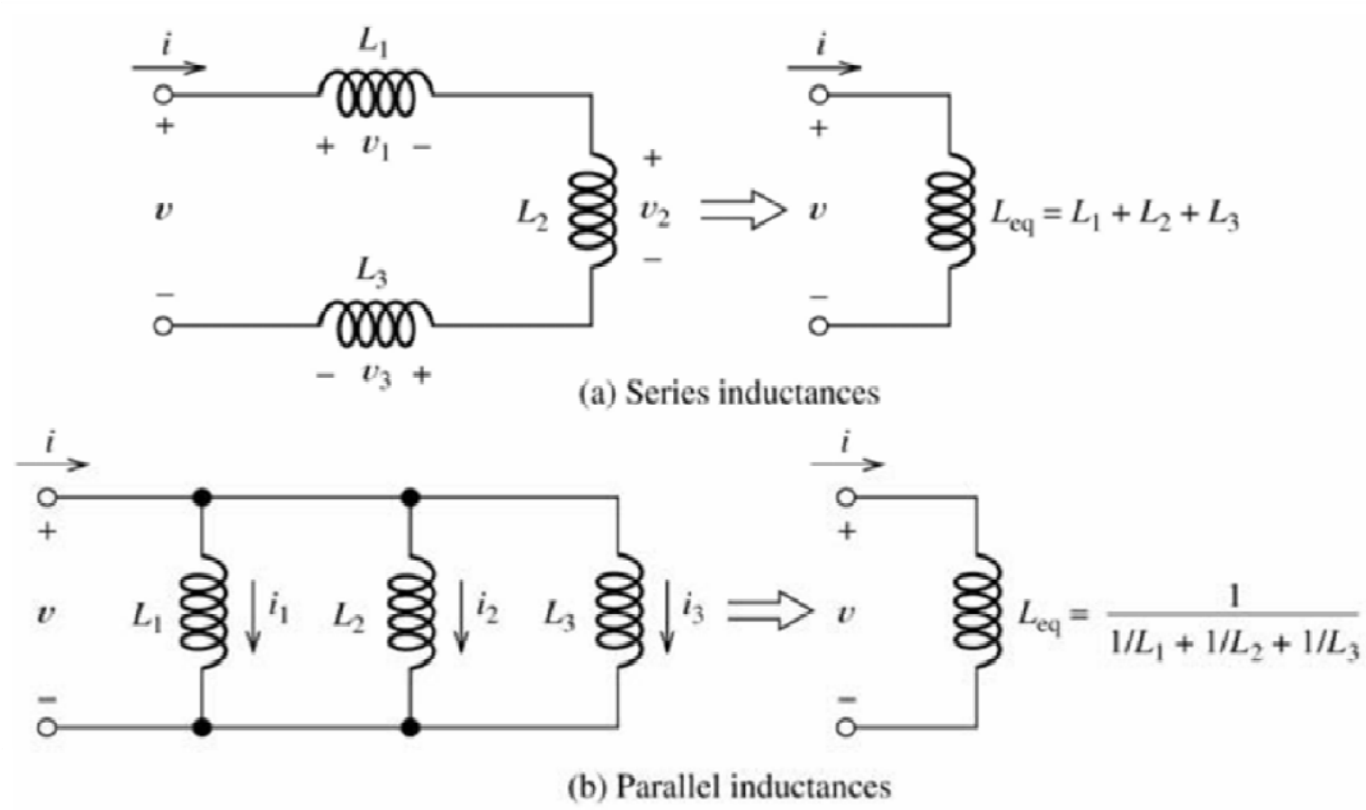
  c) ii, iv, v

  d) i, iii, v

  Answer: a

Inductors in Series and Parallel

- KVL: $V_{tot} = L1\frac{di}{dt} + L2\frac{di}{dt} + L3\frac{di}{dt} = \overbrace{(L1 + L2 + L3)}^{L_{eq}}\frac{di}{dt}$



Capacitors vs. Inductors

Capacitors	Inductors
I = C dV/dt	V = L dI/dt
Voltage cannot change instantaneously	Current cannot change instantaneously
Capacitances in parallel are combined like resistances in series and vice versa	Inductances are combined the same as resistances
Stores energy in electric field	Stores energy in magnetic field

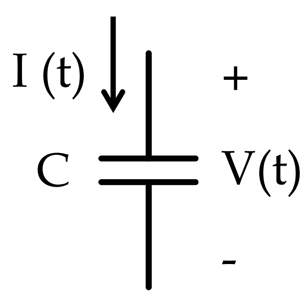
- Inductors resist a change in current
  - Current leads to the magnetic field, which takes time to increase or decrease
  - An inductor is a wire – you cannot instantaneously change current through a wire or change the flow rate of water through a pipe
  - $i(t) = \frac{1}{L}\int_{t_o}^{t} v(t)dt + i(t_o)$

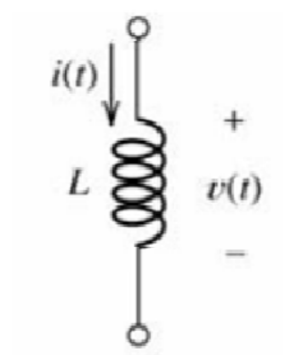
Ch. 4 Transients

- Concepts
  - Steady-state response
  - Transient response
    - Time constant
- RC and RL Circuits
- Transients – Time-varying currents and voltages due to the sudden application of sources (normally caused by switching)

DC Steady State

- **Replace capacitances with open** circuits.
- **Replace inductances with short** circuits.
- **Solve the remaining circuit.**





I = C dV/dt

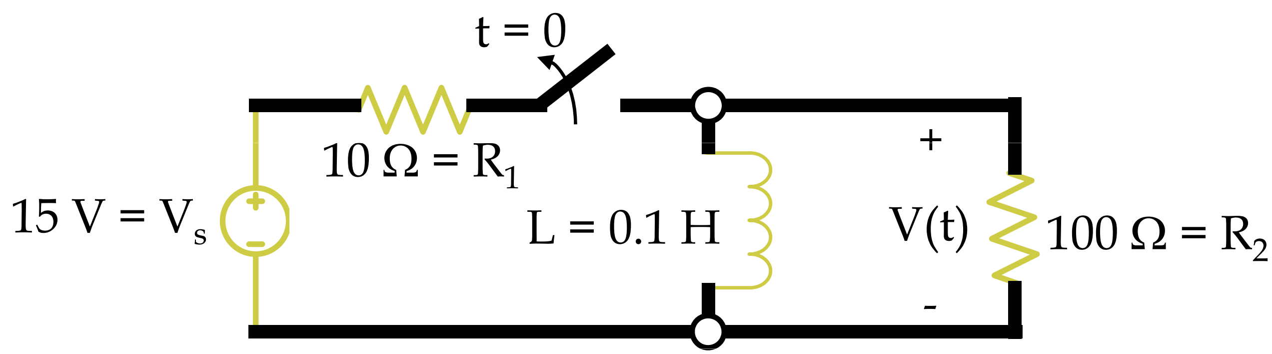
v = L di/dt

RL Circuits

Steps:

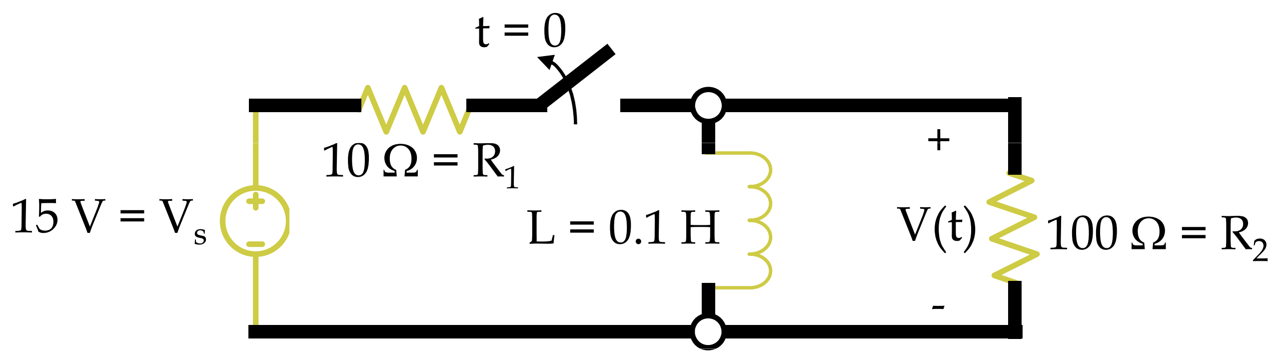
1. Find initial conditions (I.C.'s)
2. Apply KCL or KVL to get circuit equation
3. If it has integrals in it, then turn it into a pure differential equation by differentiating
4. Assume a solution of the form $K_1 + K_2 e^{st}$
5. Find K1 and s by substitution of solution into diff Eq.
6. Use I.C.'s to find K2
7. Write the final solution

Example 1



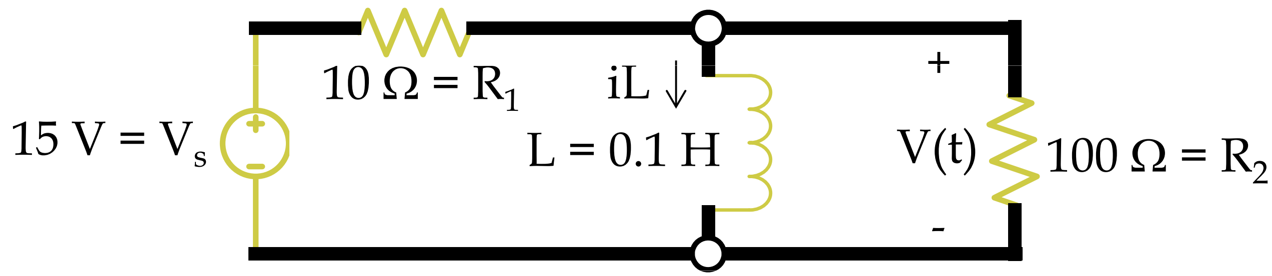
- What is the time constant after the switch opens?
- What is maximum magnitude of V(t)?
  - How does this compare to Vs?
- At what time t is V(t) ½ of its value immediately after the switch opens?

Example 1: Continued 2



- Current cannot change instantaneously in an inductor
  - How does this help us begin solving the problem?
    - The current through the inductor is same at $t = 0^-$ and $t = 0^+$
    - Reminds us that it is perfectly conducting wire that is coiled, so no voltage will appear across it under steady state conditions
- Let's find the initial conditions

Example 1: Initial S.S. Conditions



- What is the initial current iL through the inductor and V(t) for t < 0?

  a) iL = 0, V(t) = 0

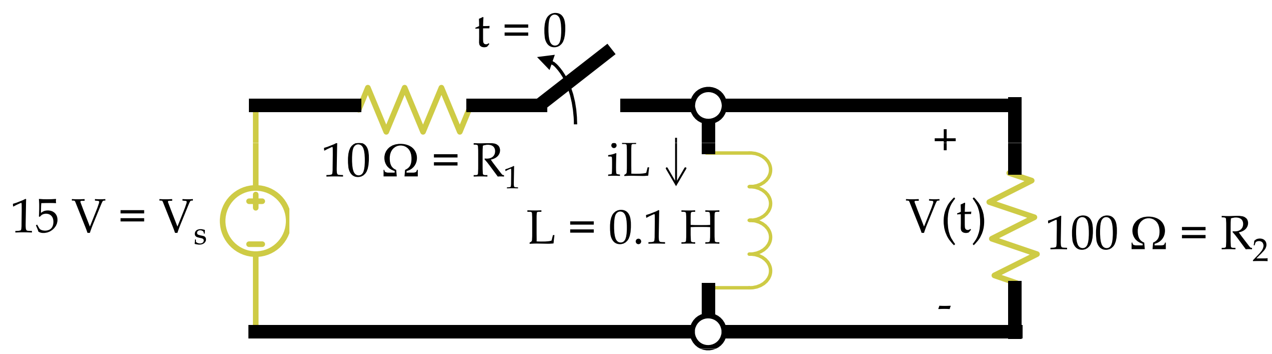
  b) iL = 0, V(t) = Vs*R2/(R1+R2)

  c) iL = Vs/(R1+R2), V(t) = Vs*R2/(R1+R2)

  d) iL = Vs/R1, V(t) = 0

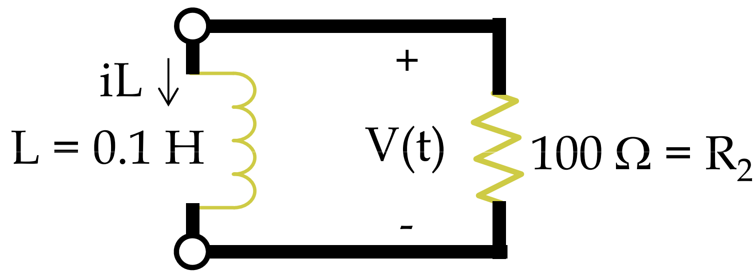
Answer: d

Example 1: What is the time constant after the switch opens?



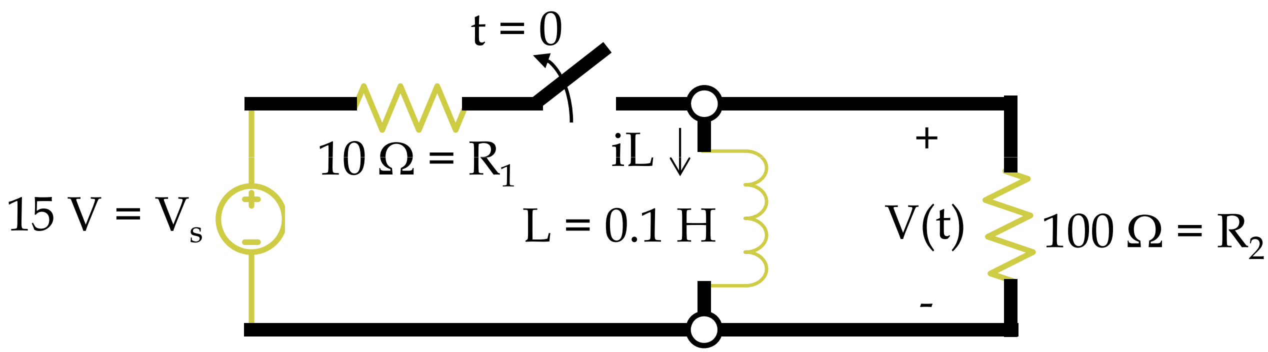
- KVL: V(t) + VL = 0
- V(t) + L di/dt = 0
- Substitute in current relationship: V(t) = i*R2
  - i*R2 + L di/dt = 0
- Simplify into a recognizable differential equation form
  - di/dt + i*R2/L = 0

Example 1: What is the time constant after the switch opens?



- 1st order differential equation: di/dt + i*R2/L = 0
  - Solution: i = ip + ih = K1 + K2e^{st}
  - Note: ip and K1 are zero: current goes to 0 as t → infinity
- Solution Form: $i_L = Ke^{-t/\tau}$
- Find K: $i(0^+)$ = Vs/R1 = Ke^0 => Vs/R1 = K
- $i_L = (Vs/R1)\, e^{-t/\tau}$
- Find τ by plugging solution into diff e.q. $\frac{-1}{\tau}Ke^{-\frac{t}{\tau}} + \frac{R2}{L}Ke^{-\frac{t}{\tau}} = 0$
- Solution $\frac{-1}{\tau} + \frac{R2}{L} = 0$ $\tau = \frac{L}{R2} = 1\ ms$

Example 1: What is maximum magnitude of V(t)?



a) -150 V
b) 5 V
c) 15 V
d) 10 V

- Answer: a
- iL = Vs/R1 at $t = 0^-$ and $t = 0^+$
- V(t) = -iL* R2 = -Vs*R2/R1 = -15 V * 100/10 = -150 V
- ***It is 10 times the magnitude of Vs!!!***

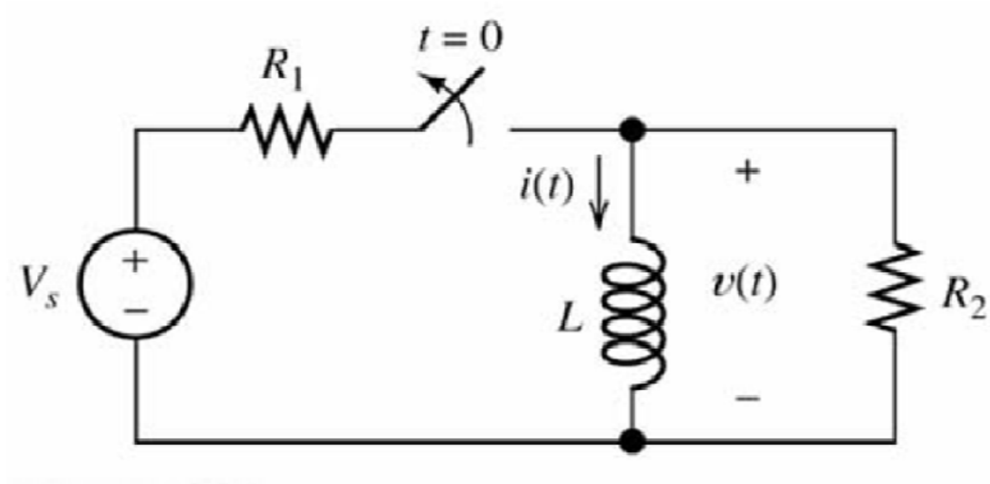
Example 1: At what time t is V(t) ½ of its value immediately after the switch opens?

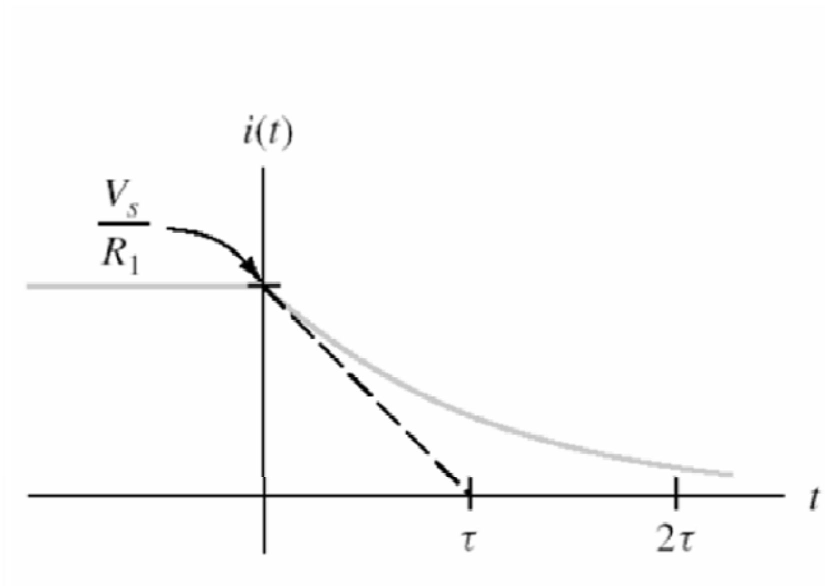
$$V(t) = L\frac{di}{dt} = L\ \frac{d(Ke^{-t/\tau})}{dt} = \frac{-LK}{\tau}e^{-t/\tau}$$

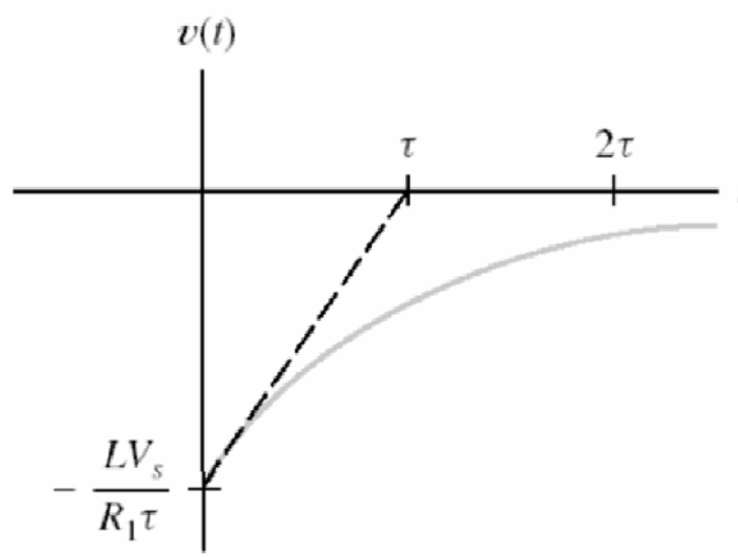
$$\frac{V2}{V1} = \frac{\frac{LK}{\tau}e^{-t2/\tau}}{\frac{LK}{\tau}e^{-t1/\tau}} \implies \frac{1}{2} = e^{\frac{-t2}{\tau}} \implies ln(\frac{1}{2}) = \frac{-t2}{\tau} \implies \quad t2 = 0.693\ ms$$

Note: t1 = 0

Current and Voltage for Example 1







Applications of RL and RC Circuits

- Capacitors and Inductors do not behave the same at all frequencies
  - Capacitor
    - In series – blocks low frequencies and acts as short circuit for high frequencies
    - In parallel – blocks high frequencies
- High Pass Filter
    - High frequency portion of signal passes through, blocks low frequency component
- Low Pass Filter
- Differentiator/Integrator
- Location of capacitor/inductor changes which type of circuit it is