EELE 250: Circuits, Devices, and Motors

Lecture 10

Assignment Reminder

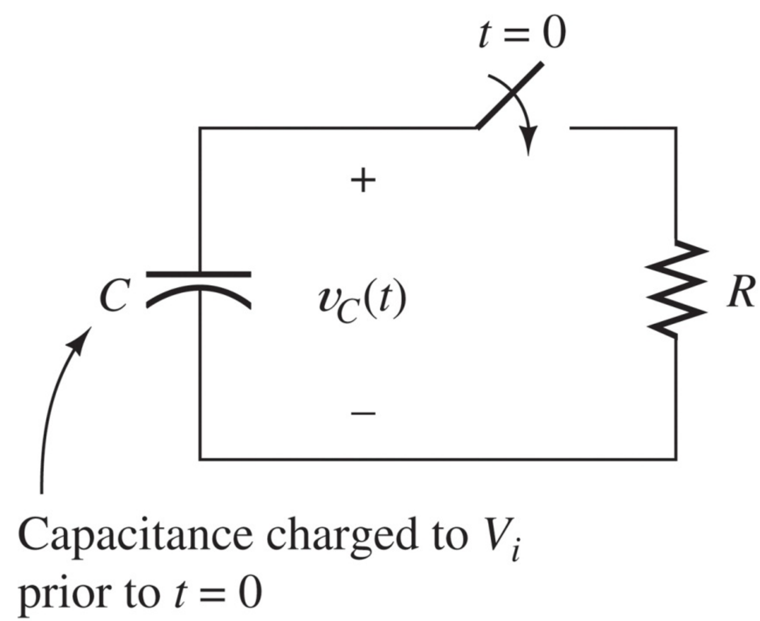
- Read 4.1 - 4.3 AND 5.1 - 5.4
- Practice Problems:
  - P3.60, 3.64, 3.72
  - P4.3, 4.8, 4.9, 4.23, 4.37, 4.38
- D2L Quiz #4 is due by 11AM on Wednesday, Sept. 25. Quiz #5 will be posted on Wednesday, and it is due by 11AM on Monday, Sept. 30.

Transients with C and L

- Transient analysis: it takes time to change voltage on a capacitor or change current in an inductor
- Analysis uses node voltage or mesh current analysis
- Typically need to consider the circuit just before t=0 (t<0 and after t=0 (t=0+)
- Cannot instantly change inductor current
- Cannot instantly change capacitor voltage

Discharging a capacitor

- Consider a capacitor, *C*, charged to some initial voltage, v_i
- At t=0, a resistance *R* is connected across the capacitor: current will start to flow, because V_i across *R* (Ohm's Law).



Discharge current

- Current in capacitor: $-C\frac{dv}{dt}$
- Current in resistor: v/R
- Node equation: $C\frac{dv}{dt}+\frac{v}{R}=0$
- Or in standard form: $\frac{dv_C(t)}{dt}+\left(\frac{1}{RC}\right)v_c(t)=0$

Discharge current (cont.)

$$\frac{dv_c(t)}{dt} + \left(\frac{1}{RC}\right) v_c(t) = 0$$

- Solution of differential equation is an exponential: $v_c(t) = Ke^{st}$

  So $sKe^{st} + \left(\frac{1}{RC}\right) Ke^{st} = 0$

- Using initial conditions: $v_c(0) = V_i$

  $s = -\frac{1}{RC}$ and $K = V_i$

Discharge Current (cont.)

- In exponent, $s \cdot t$ must be dimensionless, so $s = -\frac{1}{RC}$ implies *RC* has units of *seconds*.
- *RC* is called the *time constant*.

- The bigger the *RC* time constant, the slower the discharge time

Charging a capacitor

- Node equation for *RC* switched in series with voltage source:
- $C\frac{dv_c(t)}{dt} + \frac{v_c(t) - V_s}{R} = 0$

  $v_c(t) = K_1 + K_2 e^{st}$
- General solution:

$$v_c(t) = v_\infty + (v_0 - v_\infty)e^{-t/RC}$$

$$\tau = \text{time constant} = RC$$

Inductor transient analysis

- Procedure similar to capacitor analysis, except with differential equation involving inductor voltage in terms of inductor current
- General solution:

$$i_L(t) = i_\infty + (i_0 - i_\infty)e^{-t/(L/R)}$$

$$\tau = \text{time constant} = \frac{L}{R}$$

General Procedure for RL and RC

- Identify initial conditions:

  i_0 in inductor; v_0 for capacitor
- Identify final conditions:

  i_∞ for inductor; v_∞ for capacitor
- Identify equivalent resistance (Thevenin) "seen" by inductor or capacitor.
- Compute time constant:

  L/R_{eq} for inductor; $R_{eq} \cdot C$ for capacitor
- Then apply to general equation!

Summary and Review

- Capacitor and inductor transient analysis uses the same KVL and KCL principles we learned in Chapter 2, except with C dV/dt and L di/dt included along with Ohm's Law.
- Standard solution form: $K_1 + K_2\, e^{st}$
- General solutions:

$$v_c(t) = v_\infty + (v_0 - v_\infty)e^{-t/RC}$$

$$i_L(t) = i_\infty + (i_0 - i_\infty)e^{-t/(L/R)}$$