Problem 6.18

(a) $\mathcal{H}(\hat{\omega}) = \mathcal{H}_1(\hat{\omega})\mathcal{H}_2(\hat{\omega})$

$$\mathcal{H}_2(\hat{\omega}) = 1 - e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}}$$

$$\mathcal{H}_1(\hat{\omega}) = 1 + 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$$

Multiply:

$$\mathcal{H}(\hat{\omega}) = 1 - e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}} + 2e^{-j\hat{\omega}} - 2e^{-j2\hat{\omega}}$$
$$+2e^{-j3\hat{\omega}} - 2e^{-j4\hat{\omega}} + e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}} - e^{-j5\hat{\omega}}$$

$$\mathcal{H}(\hat{\omega}) = 1 + e^{-j\hat{\omega}} - e^{-j4\hat{\omega}} - e^{-j5\hat{\omega}}$$

MANY TERMS CANCEL OUT

(b) $h[n] = \delta[n] + \delta[n-1] - \delta[n-4] - \delta[n-5]$

(c) The polynomial coefficients of $\mathcal{H}(\hat{\omega})$ define $\{b_k\}$ as $\{1, 1, 0, 0, -1, -1\}$. Use $\{b_k\}$ as filter coefficients:

$$y[n] = x[n] + x[n-1] - x[n-4] - x[n-5]$$

Problem 7.3

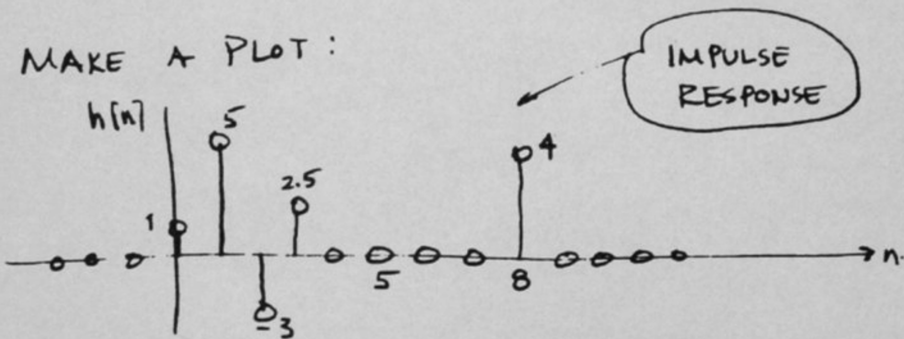
(a) $y[n] = x[n] + 5x[n-1] - 3x[n-2] + \frac{5}{2}x[n-3] + 4x[n-8]$

$$H(z) = 1 + 5z^{-1} - 3z^{-2} + \frac{5}{2}z^{-3} + 4z^{-8}$$

(b) when $x[n] = \delta[n]$, you can substitute.

$$h[n] = \delta[n] + 5\delta[n-1] - 3\delta[n-2] + \frac{5}{2}\delta[n-3] + 4\delta[n-8]$$

MAKE A PLOT:



NOTE:

The difference equation can be written as:

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

Then the impulse response will just take on the values given by the $\{b_k\}$

$\therefore$ $h[0] = b_0$, $h[1] = b_1$, $h[2] = b_2$, ... etc.

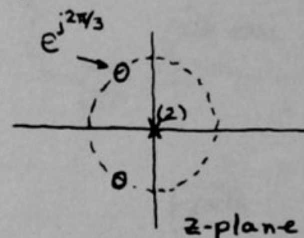
Problem 7.4

(a) use filter coeffs: $H(z) = \frac{1}{3} + \frac{1}{3}z^{-1} + \frac{1}{3}z^{-2}$

(b) Use positive powers to extract poles and zeros

$$H(z) = \frac{1}{z^2}\left(\frac{1}{3}z^2 + \frac{1}{3}z + \frac{1}{3}\right)$$

TWO POLES AT $z=0$



zeros at

$$z = \frac{-1 \pm \sqrt{1-4}}{2} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$

zeros: $1e^{\pm j2\pi/3}$

(c) $\mathcal{H}(\hat{\omega}) = H(e^{j\hat{\omega}}) = H(z)\big|_{z=e^{j\hat{\omega}}}$

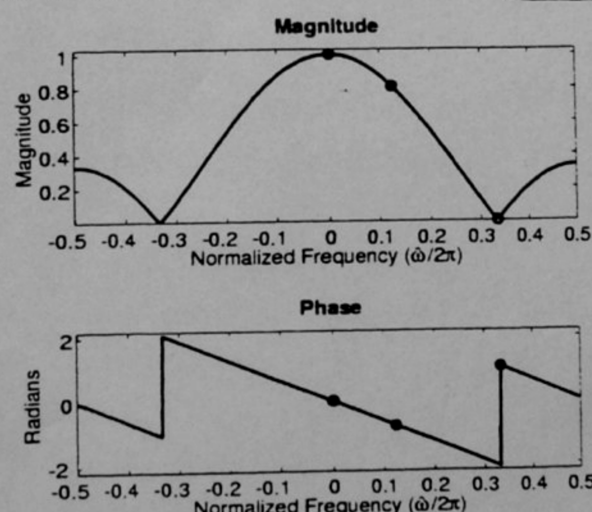
$$= \frac{1}{3} + \frac{1}{3}e^{-j\hat{\omega}} + \frac{1}{3}e^{-j2\hat{\omega}} = \frac{1}{3}e^{-j\hat{\omega}}\left(e^{j\hat{\omega}} + 1 + e^{-j\hat{\omega}}\right)$$

$$= e^{-j\hat{\omega}}\left(\frac{1+2\cos\hat{\omega}}{3}\right)$$

ANOTHER FORMULA:

$$\mathcal{H}(\hat{\omega}) = e^{-j\hat{\omega}}\left(\frac{\sin(3\hat{\omega}/2)}{3\sin(\hat{\omega}/2)}\right)$$

(d) use MATLAB



Problem 7.4 (more)

(e) Use Linearity & Frequency response at $\hat{\omega}=0$, $\hat{\omega}=\pi/4$ and $\hat{\omega}=2\pi/3$. These are marked on the plots of the frequency response.

$$y[n] = 4\mathcal{H}(0) + |\mathcal{H}(\pi/4)|\cos\left(\tfrac{\pi}{4}n - \tfrac{\pi}{4} + \angle\mathcal{H}(\pi/4)\right)$$
$$-3\underbrace{|\mathcal{H}(2\pi/3)|}_{=0}\cos\left(\tfrac{2\pi}{3}n + \angle\mathcal{H}(2\pi/3)\right)$$

$$\mathcal{H}(0) = \tfrac{1}{3}+\tfrac{1}{3}+\tfrac{1}{3} = 1$$

$$\mathcal{H}(\pi/4) = e^{-j\pi/4}(1+2\sqrt{2}/2)/3 = \frac{1+\sqrt{2}}{3}e^{-j\pi/4} = 0.8047e^{-j\pi/4}$$

$\mathcal{H}(2\pi/3) = 0$ because $H(z)=0$ at $z=e^{\pm j2\pi/3}$

$$\therefore\ y[n] = 4 + 0.8047\cos\left(\tfrac{\pi}{4}n - \tfrac{\pi}{2}\right)$$

Problem 7.12

(a) $H(z) = (1-z^{-1})(1+z^{-2})(1+z^{-1})$ MULTIPLY OUTER FACTORS

$= (1-z^{-2})(1+z^{-2}) = 1-z^{-4}$

$\therefore \; y[n] = x[n] - x[n-4]$

(b) $H(e^{j\hat{\omega}}) = H(z)\big|_{z=e^{j\hat{\omega}}} = 1 - e^{-j4\hat{\omega}}$

(c) $H(e^{j\hat{\omega}}) = e^{-j2\hat{\omega}}(e^{+j2\hat{\omega}} - e^{-j2\hat{\omega}})$

$= 2j\, e^{-j2\hat{\omega}} \sin 2\hat{\omega} = (2\sin 2\hat{\omega})\, e^{j(\pi/2 - 2\hat{\omega})}$

MAG: $2\sin 2\hat{\omega}$ ALTHOUGH THIS HAS A SIGN CHANGE FOR $\hat{\omega} < 0$

PHASE: $\pi/2 - 2\hat{\omega}$

(d) BLOCK WHEN $H(e^{j\hat{\omega}}) = 0$

$\therefore$ SOLVE $2\sin 2\hat{\omega} = 0$

$\Rightarrow \hat{\omega} = 0, \pi/2, \pi, -\pi/2$

(e) Need $H(e^{j\pi/3})$ because that is the frequency of the input.

$H(e^{j\pi/3}) = \left(2\sin\frac{2\pi}{3}\right) e^{j(\pi/2 - 2\pi/3)}$

$= 2\left(\frac{-\sqrt{3}}{2}\right) e^{j(3\pi/6 - 4\pi/6)}$

$= -\sqrt{3}e^{-j\pi/6} = \sqrt{3}e^{j\pi}e^{-j\pi/6} = \sqrt{3}\, e^{j5\pi/6}$

$\therefore$ OUTPUT IS: $y[n] = \sqrt{3}\cos\left(\frac{\pi n}{3} + \frac{5\pi}{6}\right)$