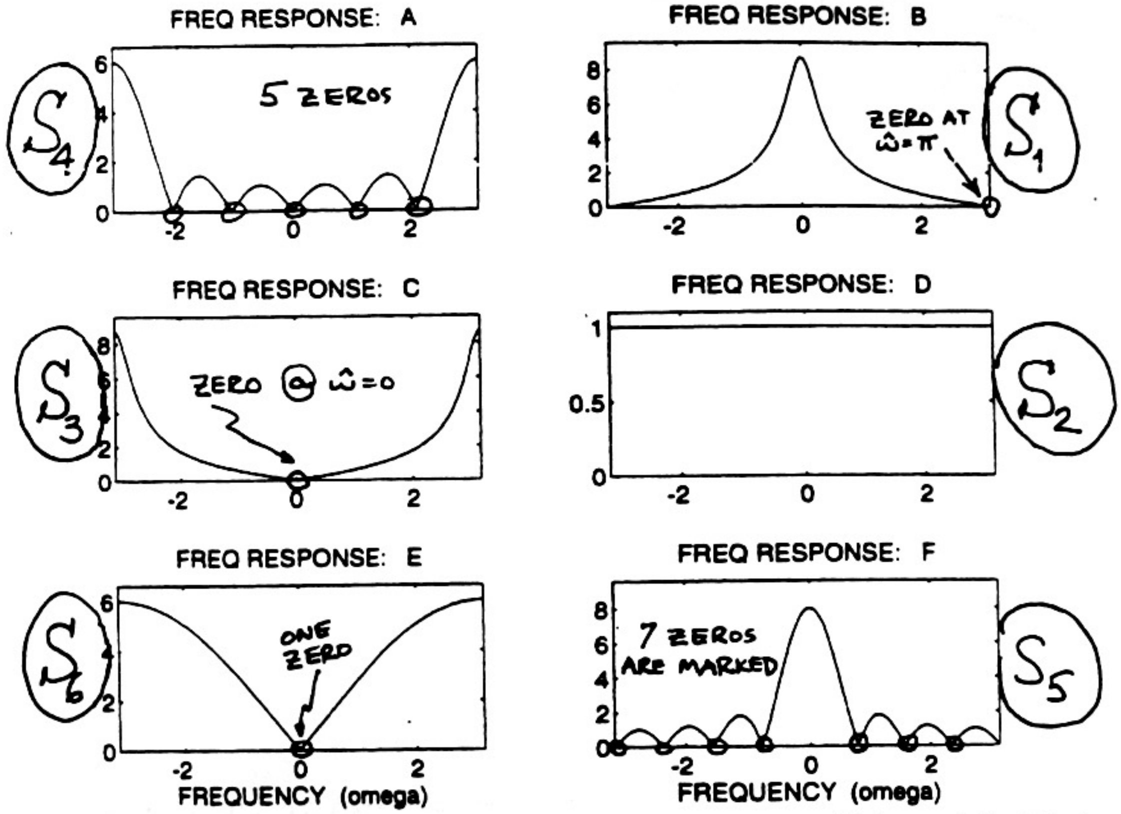
Problem 8.14



For each of the frequency response plots (A, B, C, D, E, F), determine which one of the following systems (specified by either an $\mathcal{H}(z)$ or a difference equation) matches the frequency response.

S_1 : $y[n] = 0.77y[n-1] + x[n] + x[n-1]$ ← LOWPASS w/ ZERO AT $\hat{\omega} = \pi$

S_2 : $y[n] = 0.77y[n-1] + 0.77x[n] - x[n-1]$ → $H(z) = \dfrac{0.77 - z^{-1}}{1 - 0.77z^{-1}}$ is ALL-PASS

S_3 : $\mathcal{H}(z) = \dfrac{1 - z^{-1}}{1 + 0.77z^{-1}}$ ← HIGH-PASS w/ ZERO AT $\hat{\omega} = 0$

S_4 : $\mathcal{H}(z) = 1 - z^{-1} + z^{-2} - z^{-3} + z^{-4} - z^{-5}$ ← "Dirichlet" HIGH PASS w/ 5 ZEROS ON U.C.

S_5 : $y[n] = \sum_{k=0}^{7} x[n-k]$ ← "Dirichlet" LOW PASS w/ 7 ZEROS ON U.C.

S_6 : $\mathcal{H}(z) = 3 - 3z^{-1}$ HIGH-PASS w/ 1 ZERO

S_7 : $y[n] = x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4] + x[n-5]$

→ "Dirichlet" LOWPASS w/ 5 ZEROS ON U.C.

Problem 8.16

General comments:

(1) Pole-Zero plots #1 & #2 are for FIR filters because all poles are at $z=0$

(2) Freq. Responses A & C have many zeros which correspond to zeros on UNIT CIRCLE

(3) Freq Resp C is a Lowpass filter so its pole-zero plot should have no zero near $z=0$

$\therefore$ PZ #1 $\longleftrightarrow$ C

#2 $\longleftrightarrow$ A

(4) Freq. Resp. D has prominent peaks at $\pm\pi/2 \Rightarrow$ poles near $z = e^{\pm j\pi/2} = \pm j$.

$\therefore$ #6 $\longleftrightarrow$ D.

(5) Freq Resp. E has several peaks each caused by different pole-pairs. Notice peaks at $\hat{\omega} \approx \pm\pi/6$.

$\therefore$ #3 $\longleftrightarrow$ E

(6) Freq. Response B is Highpass filter which needs a pole near $z=-1$

$\therefore$ #5 $\longleftrightarrow$ B.

Problem 9.2

(a) $X_0[k] = \sum_{n=0}^{9} x_0[n] e^{-j(2\pi/10)nk} = x_0[0] e^{-j0} = 1$

only non-zero for n=0

(b) $X_1[k] = \sum_{n=0}^{9} 1\, e^{-j(2\pi/10)nk}$

$$= \begin{cases} 10 & \text{if } k=0 \\ \dfrac{1-e^{-j(2\pi/10)10k}}{1-e^{-j2\pi/10\,k}} = \dfrac{1-1}{\text{denom}} = 0 & \text{if } k=1,2,\dots 9 \end{cases}$$

(c) $X_2[k] = \sum_{n=0}^{9} x_2[n] e^{-j(2\pi/10)nk} = e^{-j(2\pi/10)4k}$

non-zero only for n=4

$X_2[k] = e^{-j(4\pi/5)k}$ for $k=0,1,2,\dots 9$

(d) $X_3[k] = \sum_{n=0}^{9} e^{j2\pi n/5} e^{-j(2\pi/10)nk}$

$$= \sum_{n=0}^{9} e^{j(2\pi/10)(2n-nk)}$$

$$= \begin{cases} 10 & \longleftarrow \text{when } k=2 \\ \dfrac{1-e^{j(2\pi/10)10(2-k)}}{1-e^{j(2\pi/10)(2-k)}} = \dfrac{1-1}{\text{denom}} = 0 & \text{when } k\neq 2 \end{cases}$$

$$X_3[k] = \begin{cases} 10 & \text{for } k=2 \\ 0 & \text{for } k=0,1,3,4,5,6,7,8,9 \end{cases}$$

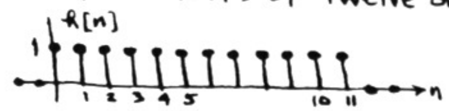
Problem 9.7

The 12-pt running sum filter has filter coeffs:

$\{b_k\} = \{1, 1, 1, \ldots 1\}$ (Twelve ones)

(a) The impulse response $h[n]$ consists of twelve ones.

$$h[n] = \sum_{k=0}^{11} \delta[n-k]$$



(b) The frequency response contains a "Dirichlet" form: $H(e^{j\hat{\omega}}) = e^{-j11\hat{\omega}/2} \dfrac{\sin(6\hat{\omega})}{\sin(\hat{\omega}/2)}$

When $x[n] = e^{j\pi n/4}$, the output is:

$$y_b[n] = H(e^{j\pi/4})e^{j\pi n/4}$$

$$e^{-j11\pi/8} \frac{\sin(3\pi/2)}{\sin(\pi/8)}$$

$H(e^{j\pi/4}) = 2.613e^{-j3\pi/8}$ ← ANGLE = $-67.5°$ = -1.178 rads

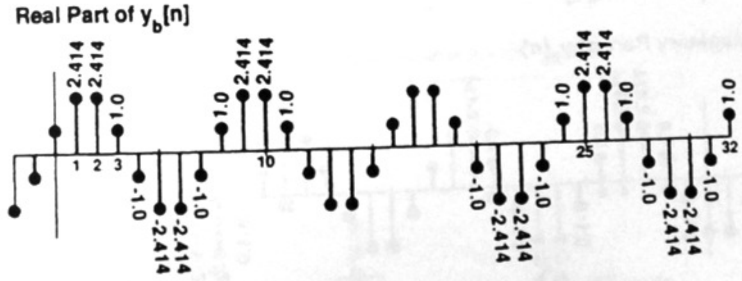
$$y_b[n] = 2.613\, e^{j(\pi n/4 - 3\pi/8)}$$

(c) When the input signal $x[n]$ has a starting time at $n=0$, the filter exhibits a "transient" over the region $0 \le n < 11$. For $n \ge 11$ the output can be determined by using the frequency response, as in part (b) — we just take the real part.

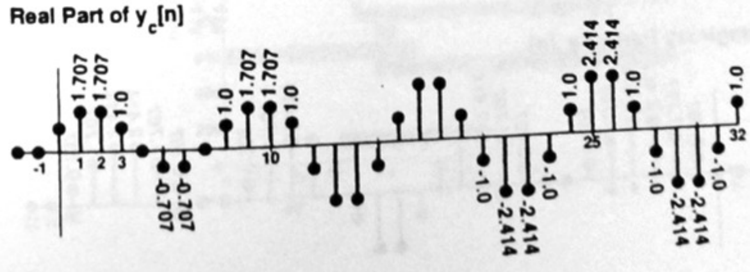
$y_c[n] = 2.613 \cos\left(\frac{\pi}{4}n - \frac{3\pi}{8}\right)$ for $n \ge 11$

$y_c[n] = 0$ ———————— for $n < 0$

Real Part of $y_b[n]$



Real Part of $y_c[n]$



Problem 9.7 (more)

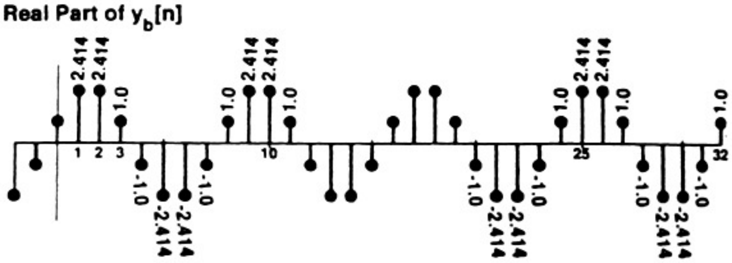
(d) In this case, there are two "transient" regions
$0 \le n < 11$ and $20 \le n < 31$.
For $n < 0$ and $n \ge 31$ the output is zero.
In the "steady-state" region $11 \le n < 20$, we get the same answer as part (b).

$$y_d[n] = 0 \quad \text{for } n < 0 \text{ or } n \ge 31$$

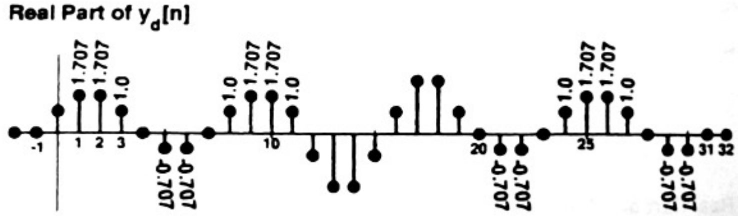
$$y_d[n] = 2.613\, e^{j(\pi n/4 - 3\pi/8)} \quad \text{for } 11 \le n < 20$$

The real & imaginary parts of $y_d[n]$ are plotted.

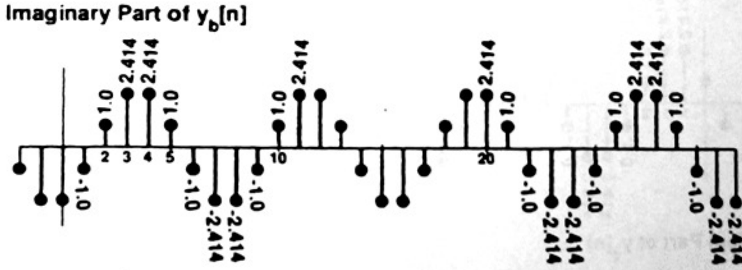
Real Part of $y_b[n]$



Real Part of $y_d[n]$



Imaginary Part of $y_b[n]$



Imaginary Part of $y_d[n]$