# EE 477
# Digital Signal Processing

5a

FIR Discrete-Time Systems

# Discrete-Time System

- Input and Output are discrete-time sequences:

$$y[n] = F(x[n])$$

- Some systems depend only on the current input:

$$y[n] = 5x[n]$$

$$y[n] = 3(x[n])^2$$

# Discrete-Time System (cont.)

- A system may depend on the current and past inputs:

$$y[n] = 5x[n] + 2x[n-1] - 6x[n-2]$$

- Or on past outputs:

$$y[n] = y[n-1] + 2y[n-2] - 3y[n-3]$$

# Discrete-Time System (cont.)

- A system can also depend on *future* inputs, but this won't work in real time:

$$y[n] = 5x[n] + 2x[n+1] - 6x[n+2]$$

- Or any of the infinite combinations and permutations.

# Difference Equations

- Many systems can be represented by an input-to-output mathematical expression of the form:

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

- This is a *difference* equation. It can fully describe the system.

# Causal System

- A *causal* system depends only on the current and past inputs.

- A real time system must be causal.

- Non-causal systems may be OK if a time delay is allowable.

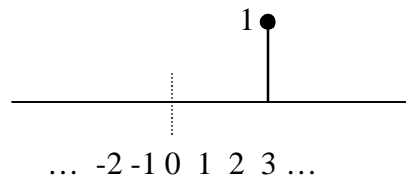# Example: Average

$$y[n] = 0.5*x[n] + 0.5*x[n-1]$$

- If x[n] changes slowly, x[n] $\approx$ x[n-1], and y[n] is approximately equal to x[n] .
- If x[n] changes rapidly, x[n] $\neq$ x[n-1], and y[n] is small.

# Unit Sample Sequence

- Unit sample, or unit impulse:

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & else \end{cases}$$

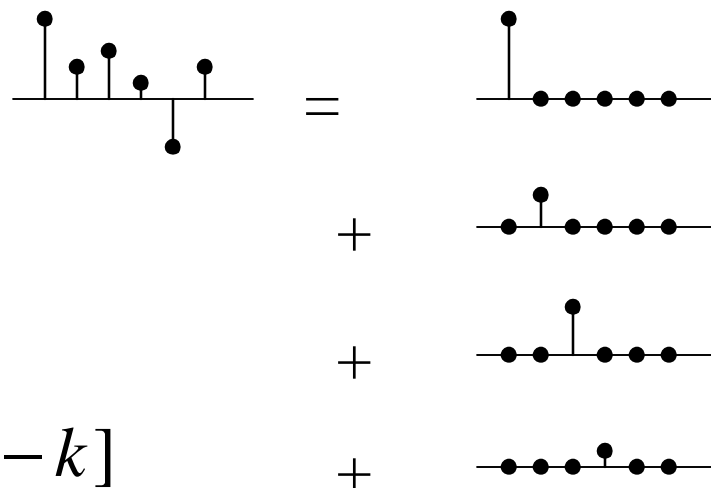- A shifted impulse: $\delta[n-3]$



… -2 -1 0  1  2  3 …

# Unit Sample Sequence (cont.)

- Notice that a sequence can be represented as a sum of shifted and weighted unit samples:

$$x[n] = \sum_k x[k] \cdot \delta[n-k]$$

# Finite Impulse Response

- Discrete-time systems can be characterized by the *impulse response*.

- Apply a single non-zero input sample (a digital *impulse*) and observe the output.

- If the output becomes exactly zero (and stays that way) sometime after the impulse, the system has a *finite-length impulse response*, or is *FIR* for short.

# FIR (cont.)

unit sample sequence $\delta[n]$



… -2 -1 0 1 2 …

FIR

… -2 -1 0 1 2 3 4 …
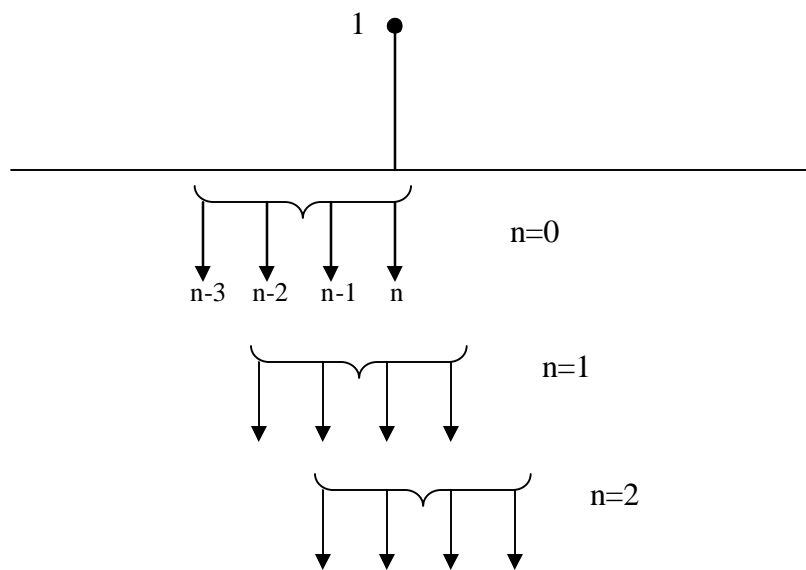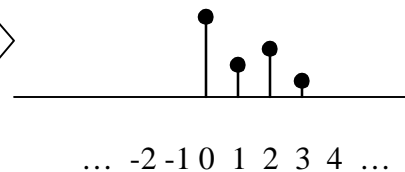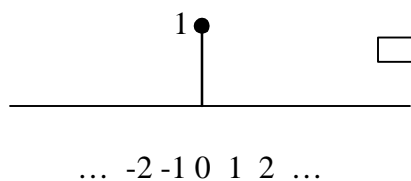
$$y[n]=a \cdot x[n]+b \cdot x[n-1]+c \cdot x[n-2]+d \cdot x[n-3]$$

n-3  n-2  n-1  n

n=0

n=1

n=2

$y[0]=a$

$y[1]=b$

$y[2]=c$

$y[3]=d$

For FIR: unit sample response is just the sequence of coefficients

# Discrete-Time Convolution

- Can express FIR system as:

$$y[n] = \sum_{k=0}^{M} h[k] \cdot x[n-k]$$

Unit sample response

- For particular *n,* overlap *h* in reverse and sum the products